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Non-equilibrium length in granular fluids: From experiment to fluctuating hydrodynamics

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Abstract – Velocity correlations in a quasi-2D driven granular fluid are studied in experiments and numerical simulations. The transverse velocity structure factor reveals two well-defined energy scales, associated with the external “bath temperature” T_b and with the internal granular one, $T_g < T_b$, relevant at large and small wavelengths, respectively. Experimental and numerical data are discussed within a fluctuating hydrodynamics model, which allows one to define and measure a non-equilibrium coherence length, growing with density, that characterizes order in the velocity field.

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The study of collective phenomena in complex systems is a central issue in statistical mechanics. In particular, the degree of *order* emerging in such systems is customarily investigated by studying the behaviour of correlation functions [1]. However, in many systems which display interesting collective phenomena interactions are *non-conservative* and a stationary state is achieved through a continuous and homogeneous energy injection acting on every single particle: this is the case, for instance, of homogeneously vibrated granular materials as well as of the so-called “active fluids” including swarms of bacteria, birds, insects, fishes, pedestrians and even artificial motors [2–4]. In all those systems, the study of correlations requires tools beyond the standard equilibrium ones.

The emergence of order out of equilibrium is studied here in a granular system [5,6], where inelastic collisions induce cross-correlations between particles velocities. The behaviour of such correlations is customarily described with fluctuating hydrodynamics (FHD), which relies on a separation of time and length scales in the system, together with a suitable model for noise. Assessing the limits of validity of granular hydrodynamics is non-trivial [7,8], but in several situations it has proven to be useful in explaining the behaviour of large-scale correlations in granular fluids [9,10]. In a more general context, the existence of non-trivial correlations emerging

from non-equilibrium assumptions was already noticed in [11]. Recently, FHD also provided a prediction for the slowing-down of the dynamics, with increasing packing fraction, in granular fluids [12,13], a phenomenon which has a well-known counterpart in elastic systems [14]. The above-mentioned and similar results [15], which clearly show the existence of a time scale growing with the density, suggest to look also for a growing length scale.

In this paper we show the existence of a finite correlation length ξ in the velocity field, peculiar to a non-conservative fluid. Velocities correlations are investigated studying the velocity structure factor measured from experiments and numerical simulations. Data are discussed within a FHD model, allowing us to extract a length scale ξ and confirming that its associated order is a peculiarity of the out-of-equilibrium regime. The behaviour of ξ is investigated at different packing fractions, showing a remarkable increase of ordering in the velocity field with growing density. The large-scale fluctuations of the velocity field suggest that a reference theoretical model where the thermostat has a finite viscous drag [16] is the most appropriate to modelize the experiments.

The experimental setup, sketched in fig. 1, consists of an electrodynamic shaker (LDS V450) which vibrates vertically (\hat{z} -axis) a horizontal rigid aluminium plate covered by a partial granular monolayer of N “steel 316” spheres, with packing fraction $\phi = \pi n \sigma^2 / 4 < 0.5$, where $n = N/\text{Area}$ is the number density and the diameter σ of spheres is 4 mm. The plate is circular and has slowly

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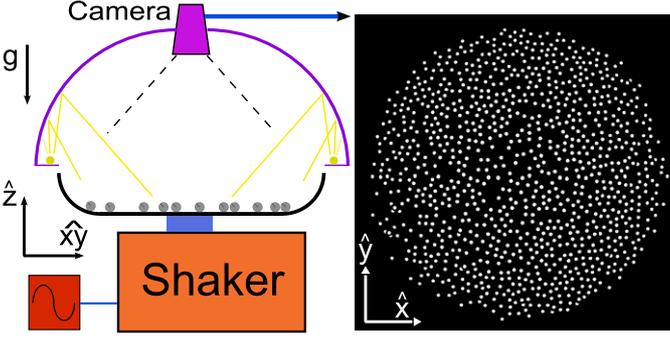


Fig. 1: (Color online) Schematic of the experimental setup.

rising boundaries in order to prevent highly dissipative head-on collisions between spheres and walls. The radius of the central flat region is $R = 100$ mm. The surface of the plate is rough [17–19], transferring part of the vertical momentum to the horizontal ($\hat{x}\hat{y}$ plane) motion of the spheres. The plate is roughened by stainless-steel sand blasting with a granulometry in the range $[100\text{--}500]\mu\text{m}$. Collisions among the spheres are also responsible for $\hat{z} \rightarrow \hat{x}\hat{y}$ energy conversion. From a fast camera (Mikrotron EoSens CL, up to 506 fps for 1280×1024 pixels) placed on the top of the plate, we observe the $\hat{x}\hat{y}$ motion of the spheres. Even if the container has no lid (as it was in the experiment with the rough plate in [17]), we verified by visual inspection that—at the chosen shaking parameters (see below)—particles never sit on top of each other and constitute a quasi-two-dimensional fluid. The results discussed below are obtained for sinusoidal shaking, *i.e.* the plate follows the law $z(t) = A \sin(\omega t)$, with $A\omega^2/g = 12$ and $\omega/(2\pi) = 150$ Hz, where g is the gravity acceleration, yielding a maximum velocity of the vessel $v_m \simeq 125$ mm/s. A particle tracking software identifies particles' centers of mass, up to an accuracy of $1/5$ of a pixel, and their displacement along close successive frames ($\Delta t = 1/57$ s), reconstructing particles' velocities: the analysis of the mean squared displacement $\Delta s^2(\tau) = \langle |x(t+\tau) - x(t)|^2 \rangle$ guarantees that the reconstruction is operated in the ballistic regime, *i.e.* $\Delta s^2(\tau) \sim \tau^2$ for $\tau < 2\Delta t$.

The measure of the static structure factor $S_\rho(k) = 1/V \sum_{i=1}^N \langle |\exp(-i\mathbf{k} \cdot \mathbf{r}_i)|^2 \rangle$, where \mathbf{r}_i denotes the position of each particle and V is the volume, reveals that in the range of packing fraction that we studied, *i.e.* $\phi \in [0.1, 0.42]$, the structure of the granular fluid presents small differences from its elastic counterpart (inset of fig. 2), probably due to weak clustering or inhomogeneities. The small enhancement of the structure may also be understood by realizing that—in our setup—a slight amount of vertical motion allows particles to arrange at separations even smaller than their diameter.

More peculiar is the behaviour of the velocity structure factor. In particular we study here its transverse part:

$$n^2 S_\perp(k) = V^{-1} \langle v_\perp(k) v_\perp(-k) \rangle, \quad (1)$$

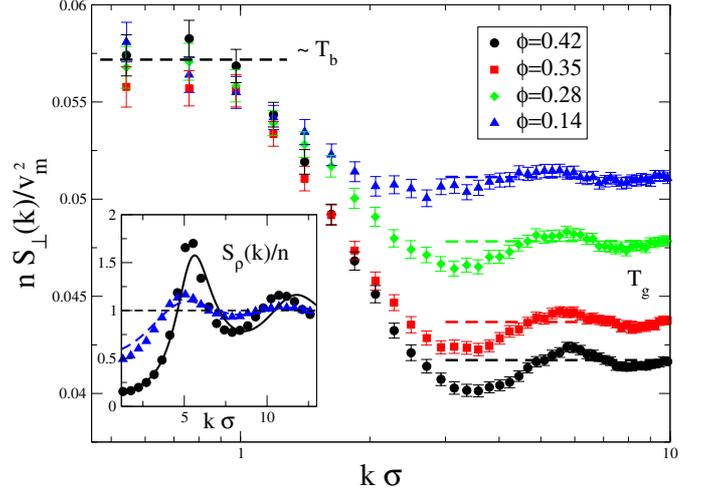


Fig. 2: (Color online) $S_\perp(k)$ measured in units of v_m^2 in experiments (where v_m is the maximum velocity of the vibrating plate), for different values of ϕ . Data represent averages over 3300 pairs of frames. The dotted lines on the right show the corresponding T_g . As a reference for the eye, we also indicate the average (among the four experiments) of T_b , which is defined in the text. Inset: $S_\rho(k)$ for $\phi = 0.14$ (\blacktriangle) and $\phi = 0.42$ (\bullet) with the analytical predictions for elastic disks [20]. In all figures the lowest wave vector is $k\sigma \geq 2k_{min}\sigma$ ($2k_{min}\sigma = 4\pi\sigma/R = 0.5$), because at smaller ones finite-size effects are dominant. The values of $S_\perp(k)$ reported are an average on moments with modulus $k \in [2n\pi/R, (n+1)2\pi/R]$ for integer $n \geq 2$.

with $v_\perp(k) = \sum_i (\mathbf{v}_i \cdot \hat{k}_\perp) \exp(-i\mathbf{k} \cdot \mathbf{r}_i)$, where \mathbf{v}_i denotes the velocity of the i -th particle and \hat{k}_\perp is the unitary vector such that $\hat{k}_\perp \cdot \hat{k} = 0$. The curves of $nS_\perp(k)$ measured in the range of packing fractions $\phi \in [0.1, 0.42]$, *i.e.* in a moderate dense fluid regime, are shown in the main frame of fig. 2.

By definition, $S_\perp(k)$ yields a measure of the energy localized on average on each mode k . In the granular system studied here, we find that a different amount of energy is concentrated on each mode, namely at stationarity $S_\perp(k)$ is a non-trivial function of k . More specifically, we observe that *two* relevant energy scales are well defined in the system: one at large k , which corresponds to the granular temperature $T_g = 1/(2N) \sum_i \langle v_i^2 \rangle$ and strongly depends on the packing fraction ϕ ; the other, at small k , is weakly dependent on ϕ . Consider that in the limit $k \rightarrow 0$, we have that, by definition, $nS_\perp(k) \rightarrow N \langle V_{cm}^2 \rangle$, with V_{cm} the velocity of the center of mass of the system. The quantity $N \langle V_{cm}^2 \rangle$ is not affected by momentum-conserving collisions: it is, therefore, related *only* to the interaction with the vibrating plate. For such a reason, we use this value as a measurement of the so-called “bath temperature” T_b . In the theoretical model introduced below this energy corresponds to the temperature of the thermostat coupled to the granular fluid. From fig. 2 it is evident that the gap between the two energy scales defined above significantly grows as the packing fraction is increased. We will show that the existence of such a gap is deeply connected with

the existence of a finite correlation length in the velocity field. The oscillations of $S_{\perp}(k)$ at high values of k are likely to be a signature of an unexpected coupling with the density modes, already observed in previous works [21].

With the aim of giving a consistent interpretation of our experimental results, we now propose an effective model for the microscopic dynamics of particles (restricted to the plane xy) and deduce from it a theoretical formula—through fluctuating hydrodynamics [21,22]—for $S_{\perp}(k)$. Our proposal is well known in the literature [16]: it mimics the action of the rough vibrating plate by means of an interaction with an effective viscous “bath”. The model is described by the following equation governing the dynamics of the i -th particle ($i \in [1, N]$):

$$\dot{\mathbf{v}}_i(t) = -\gamma_b \mathbf{v}_i(t) + \boldsymbol{\zeta}_i(t) + \mathbf{F}_i, \quad (2)$$

where \mathbf{F}_i represents the effect of particle-particle inelastic hard-core collisions with restitution coefficient α , while γ_b^{-1} is the typical interaction time with the plate and $\boldsymbol{\zeta}_i(t)$ a white noise, with zero mean and $\langle \zeta_{i,\alpha}(t) \zeta_{j,\beta}(t') \rangle = 2T_b \gamma_b \delta_{\alpha\beta} \delta(t-t') \delta_{ij}$ ($\alpha, \beta = \{x, y\}$). This kind of thermostat is well defined also in the elastic limit $\alpha \rightarrow 1$, where the fluid equilibrates to the bath temperature: $T_g = T_b$ [16].

For the above model one can apply the usual kinetic theory to study hydrodynamic fields [21]. In linearized hydrodynamics, which applies to the moderately dense regime here studied, the transverse modes $v_{\perp}(k, t)$ of the velocity field are decoupled from all the other modes. In particular, at small k they are well described—on average—by the equation $\dot{v}_{\perp}(k, t) = -(\gamma_b + \nu k^2)v_{\perp}(k, t)$, with ν the kinematic viscosity [1]. This equation contains two main sources of relaxation: the external friction parametrized by γ_b , and vorticity diffusion parametrized by ν . To analyze structure factors one needs to plug in appropriate noises. The two relaxation mechanisms described above are associated with two independent sources of randomness: external and internal noise, respectively. A rigorous evaluation of the way those noises enter the description for $v_{\perp}(k, t)$ can be found in the literature for different models [22]. Here we present a more phenomenological approach, assessed by its ability to reproduce experimental and numerical results. The idea, partly inspired by previous granular FHD theories [21], consists in keeping valid—as a first approximation—the 2nd-kind fluctuation dissipation relation (FDR) for the two relaxation-noise pairs, separately. The result can be cast in a Langevin equation for the shear mode, with one single noise with appropriate variance:

$$\begin{aligned} \dot{v}_{\perp}(k, t) &= -(\gamma_b + \nu k^2)v_{\perp}(k, t) + \eta(k, t), \\ \langle \eta(k, t) \eta(-k, t') \rangle &= 2(\gamma_b T_b + \nu k^2 T_g) \delta(t-t'), \end{aligned} \quad (3)$$

with $\langle \eta(k, t) \rangle = 0$. The amplitudes of the external and internal noises in the above equation are reasonably assumed to be proportional to T_b and T_g , respectively. The p.d.f. of $v_{\perp}(k)$ measured in experiments are Gaussian, in agreement with eq. (3), whereas those of real-space

velocities v_i show deviations from Gaussianity due to the relevance of collisions in eq. (2). From eq. (3) we immediately obtain

$$nS_{\perp}(k) = N^{-1} \langle |v_{\perp}(k)|^2 \rangle = \frac{\gamma_b T_b + \nu k^2 T_g}{\gamma_b + \nu k^2}. \quad (4)$$

At equilibrium, namely when collisions are elastic and $T_g = T_b$, equipartition between modes is perfectly fulfilled and the structure factor becomes flat, *i.e.* $S_{\perp}(k) = T_b$. Differently, in the granular case, where $T_g \neq T_b$, equipartition breaks down and from eq. (4) we have that $S_{\perp} \rightarrow T_b$ for small k and $S_{\perp} \rightarrow T_g$ for large k . The Fourier inversion of eq. (4) in two dimensions yields

$$nG_{\perp}(\mathbf{r}) = T_g \delta^{(2)}(\mathbf{r}) + (T_b - T_g) \frac{K_0(r/\xi)}{\xi^2}, \quad (5)$$

where $K_0(x)$ is the 2nd-kind modified Bessel function that, for large distances, decays exponentially,

$$K_0(r/\xi) \approx \sqrt{\frac{\pi}{2}} \frac{e^{-r/\xi}}{(r/\xi)^{1/2}}, \quad (6)$$

and we have introduced $\xi = \sqrt{\nu/\gamma_b}$. Such quantity turns out to be a *non-equilibrium* correlation length, measurable in the system *only* when non-conservative interactions are present. Indeed, at equilibrium, the second term in the r.h.s. of eq. (5) vanishes and no coherence can be identified in the velocity field fluctuations.

The saturation of $nS_{\perp}(k)$ to a finite value at small k provided from this theory is in agreement with the behaviour of experimental data observed in fig. 2, suggesting that the model with finite friction is appropriate to describe the system.

The correlation length ξ is defined through γ_b and ν , which can be obtained by studying the dynamical correlation $C_{\perp}(k, t) = \langle v_{\perp}(k, t) v_{\perp}(-k, 0) \rangle / \langle |v_{\perp}(k, 0)|^2 \rangle$. Indeed, from eq. (3), we immediately obtain $C_{\perp}(k, t) = \exp\{-(\gamma_b + \nu k^2)t\}$. Such behaviour is in good agreement with experimental data, as reported in fig. 3, where $C_{\perp}(k, t)$ is plotted for several values of ϕ (fig 3(a)) and different wave vectors k (fig. 3(b)), together with the best-fit curves. At each packing fraction, the length scale $\xi = \sqrt{\nu/\gamma_b}$ can be obtained by comparing the decay of shear modes at different k (see fig. 3(d)). A parabolic fit $\tau^{-1}(k) = \gamma_b + \nu k^2$ shows that our data are broadly consistent with the theory, but it is also important to notice that only few data points are available. In fig. 3(c) we see how the correlation length ξ , rescaled with the prediction—in the diluted approximation—for the mean free path $\lambda_0 = 1/(2\sqrt{\pi}g_2 n\sigma)$, where $g_2 = (1 - 7\phi/16)/(1 - \phi)^2$, significantly grows with the packing fraction and signals an increasing degree of order in the granular fluid. Notice that the main contribution to the growth presented in fig. 3(c) is due to the decrease of λ_0 , while ξ grows only weakly, as already visible in fig. 2, where $S_{\perp}(k)$ has a “sigmoidal” shape with an inflection

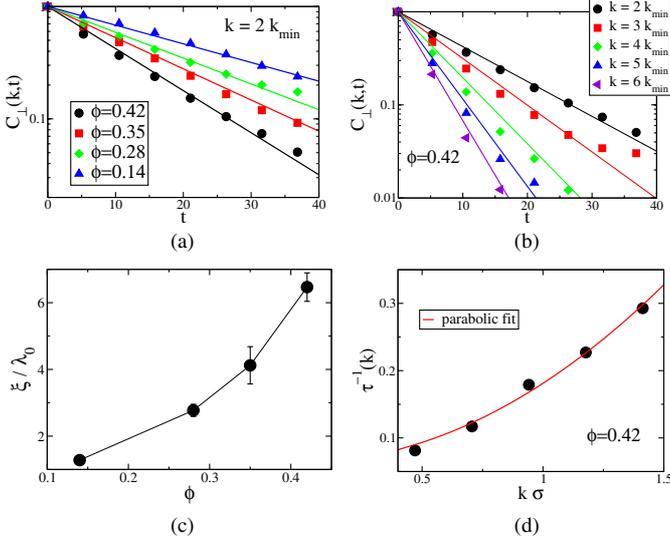


Fig. 3: (Color online) Data from experiments. Temporal decay of $C_{\perp}(k, t)$ at fixed $k = 2k_{\min}$ (a) and for different k at fixed $\phi = 0.42$ (b). The unit of times is $2\pi/\omega$. (c) Correlation length ξ , rescaled by λ_0 , plotted as function of ϕ . (d) Parabolic fit of $\tau^{-1}(k) = \gamma_b + \nu k^2$.

point almost independent of ϕ . Such an observation is consistent with the experimental measurements discussed in [17]. The growth of ξ/λ_0 denotes an increasing number of correlated particles.

The finite extent of velocity correlations in a granular fluid was already pointed out in [17]. Here we discover that it is strictly related to a saturation of $S_{\perp}(k)$ as $k \rightarrow 0$, corresponding to a well-defined energy of the “external thermostat”. Within the homogeneous cooling regime a length scale ξ was discussed in [10], but in that context it separated the stable, $k > \xi^{-1}$, from the unstable $k < \xi^{-1}$, modulations of the velocity field.

The above discussion shows that a fluctuation on the mode k decays with characteristic time $\tau(k) = 1/(\gamma_b + \nu k^2)$. This time scale has to be much larger than the microscopic time scale given by the mean collision time $\tau_c \sim \lambda_0/\sqrt{T_g}$, for the FHD with white noise to be effective. Comparing $\tau(k)$ and τ_c , we see that in our experiments such constraint is fulfilled for a narrow interval of small k . This range of validity is constrained by parameters such as the kinematic viscosity ν and the drag γ_b of the thermal bath, which are not under control in experiments.

The predictions of the linearized FHD described above are tested by simulating an interacting system of inelastic hard disks modeled by eq. (2), where γ_b enters as a fixed parameter. In particular, $\gamma_b^{-1} \sim \tau_b$ is the time scale of the thermal bath, which can be tuned in order to have smaller values of T_g . Indeed, by lowering γ_b , also ν is lowered, because, in first approximation, $\nu \sim T_g^{1/2}$ [23]. Therefore, by tuning γ_b , we can raise the shear mode decay time scale $\tau(k) = 1/(\gamma_b + \nu k^2)$, extending the range of validity of FHD, *i.e.* the interval of k values where eqs. (3) and

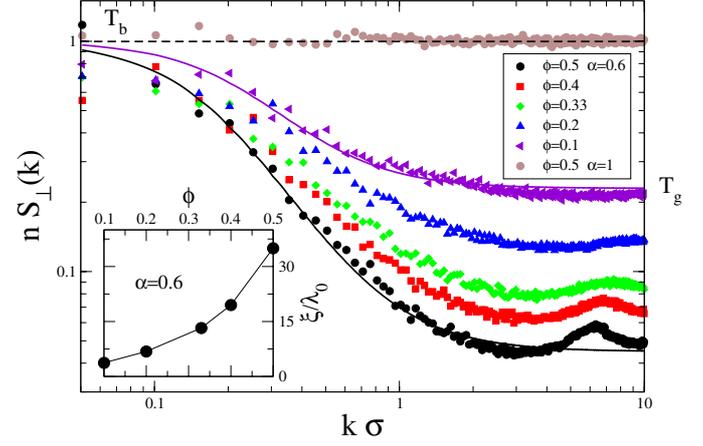


Fig. 4: (Color online) $S_{\perp}(k)$ measured in event-driven molecular-dynamics simulations, averaged over 400 frames. The solid lines are the best-fit curves via eq. (4), for $\phi = 0.42$ and $\phi = 0.14$. Inset: ξ/λ_0 as a function of ϕ .

(4) are expected to hold. We have performed 2D noisy event-driven molecular-dynamics simulations of inelastic hard disks, at different packing fractions $\phi \in [0.1, 0.5]$, with $\alpha \in [0.6, 1]$. Details and results of the simulations are reported in a separate publication [24]. We consider $N = 10000$ disks in a box of area L^2 with periodic boundary conditions and the packing fraction is varied by changing the side length of the box. The particles have diameter $\sigma = 0.01$ and unit mass and the thermal bath of eq. (2) has parameters $T_b = 1$ and $\gamma_b = 1$, while τ_c is always in the range $[0.005, 0.04]$. In fig. 4 we report our results for $S_{\perp}(k)$. A striking analogy with the experimental results of fig. 2 can be appreciated. Again, two energy scales are clearly observed: at small k all curves saturate to the value $T_b = 1$, in agreement with eq. (4). Moreover, as expected, for $\alpha = 1$ energy equipartition holds and $S_{\perp}(k) = T_b$, whereas for $\alpha \neq 1$ the structure factor at large k oscillates around the granular temperature T_g . Notice that here, by using eq. (4), we obtained a good fit of data, in a wide range of k , thus validating the FHD description (solid lines in fig. 4). The kinematic viscosity ν obtained from these fits, yields the correlation length ξ . A remarkable increase of ξ/λ_0 as a function of ϕ is found (see the inset of fig. 4). Notice also that the gap between the granular temperatures at different packing fractions and the bath temperature is strongly increased, compared to the one observed in experiments. Such a large separation between the two energy scales is consistent with an extended realm of validity of the FHD.

In conclusion we have measured the transverse velocity structure factor, and its time decay, for a 2D driven granular fluid. We have observed a striking agreement with simulations of a microscopic model with stochastic driving and viscous friction, and with a fluctuating-hydrodynamics theory with two noise sources at different temperatures $T_b \geq T_g$. A central issue of our study has

been to show the increasing extent of correlations in the velocity field, with growing density. Such a phenomenon emerges in the system only when time-reversal symmetry is broken, *i.e.* in the presence of an energy flux from the external driving to the system, due to the inequality $T_g < T_b$.

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